Conditions for Optimal Outcomes of Negotiations about Resources

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Abstract

We analyse scenarios in which individually rational agents negotiate with each other in order to agree on deals to exchange resources. We consider two variants of the framework, one where agents can use money to compensate other agents for disadvantageous deals, and another one where this is not possible. In both cases, we analyse what types of deals are necessary and sufficient to guarantee an optimal outcome of negotiations. We also show how these conditions can change depending on properties of the utility functions used by agents to represent the values they ascribe to certain sets of resources.

1 Introduction

In this paper, we analyse negotiation scenarios where agents exchange resources in order to increase their respective individual welfare. We are not concerned with any specific protocols or even strategies here, but rather with the patterns of exchanges agents could *possibly* agree on and to what extent these patterns are sufficient or necessary to guarantee optimal outcomes of negotiations. One central assumption that we *do* make with respect to the strategies that agents follow is that they are *individually rational* in the sense that they will never accept a disadvantageous deal. In the first instance, we consider the outcome of a negotiation to be optimal whenever it results in an allocation of resources with *maximal social welfare* (the sum of values ascribed by all agents to the resources they hold in that situation).

A very similar framework has been studied by Sandholm in [5] and elsewhere, mostly in the context of agents negotiating in order to reallocate tasks. In fact, much of the first half of this paper amounts to a review of [5] using a terminology appropriate for resource allocation problems (rather than task contracting). One of the central aspects of Sandholm's framework is that agents can use money to compensate other agents for accepting (otherwise) disadvantageous deals. We extend this framework here to also model negotiations over resources where no money changes hands. For these scenarios we cannot always guarantee that a negotiation will culminate in an allocation with maximal social welfare. Instead, we study conditions for outcomes that are at least Pareto optimal.

The remainder of this paper is structured as follows. In Section 2 we define our *resource allocation problems* as well as important notions such as *individual rationality*, *deal*, and *social welfare*. Section 3 discusses sufficient and necessary conditions for optimal outcomes of negotiations in the original framework *with* money and Section 4 does the same for the framework *without* money. Section 5 is a preliminary account of results for domains with specific characteristics (that is, specific classes of utility functions). Finally, we conclude with a brief discussion of ideas for future work in this area.

2 Preliminaries

In this section, we define the various components of the kind of negotiation scenario we are interested in. Negotiation occurs in a *system* $(\mathcal{A}, \mathcal{R})$ where $\mathcal{A}$ is a finite set of agents and $\mathcal{R}$ is a finite set of (discrete) resources.

Utility functions. The value an agent i ascribes to a particular set of resources R will be modelled by means of a *utility function*, that is, a function from sets of resources (subsets of $\mathcal{R}$) to real numbers. This could really be *any* such function, i.e. the utility ascribed to a set of resources is not just the sum of the values ascribed to its elements. The interesting aspect of this is that we can model the fact that utility may strongly depend on context, i.e. what other resources the agent holds at the same time. (We are going to discuss more specific classes of utility functions in Section 5.)

Allocations. An *allocation* of resources is a partitioning of the available resources amongst the agents in the system. That is, an allocation for the system $(\mathcal{A}, \mathcal{R})$ is a function A from agents in $\mathcal{A}$ to subsets of $\mathcal{R}$ such that $A(i) \cap A(j) = \{\,\}$ for $i \neq j$ and $\bigcup_{i \in \mathcal{A}} A(i) = \mathcal{R}$.

Resource allocation problems. A *resource allocation problem* is a quadruple $(\mathcal{A}, \mathcal{R}, \mathcal{U}, A_0)$ where $\mathcal{A}$ is a finite set of (at least two) agents, $\mathcal{R}$ is a finite set of resources, $\mathcal{U} = \{u_i : 2^{\mathcal{R}} \to \mathbb{R} \mid i \in \mathcal{A}\}$ is a collection of utility functions, and A_0 is an initial allocation of resources for the system $(\mathcal{A}, \mathcal{R})$.

A *solution* to a resource allocation problem is a particular allocation for the system in question that is, in a sense to be made precise, optimal.

Social welfare. A utility function u_i is a mapping from sets of resources to numerical values. The utility of an agent i for a given allocation A is the value of its utility function for the set of resources it holds in that situation. We abbreviate $u_i(A) = u_i(A(i))$.

The *social welfare* $sw(A)$ of the agent society as a whole for a given allocation A is defined as the sum of the values of the utility functions of all the agents in the society for that allocation:[1]

$$sw(A) \quad = \quad \sum_{i \in \mathcal{A}} u_i(A)$$

Rather unsurprisingly, we say that an allocation A has *maximal social welfare* for a given system $(\mathcal{A}, \mathcal{R})$ iff there is no other allocation A' for that system such that $sw(A) < sw(A')$ holds. Maximal social welfare is our first optimality criterion (a second one will be discussed in Section 4).

Deals. Agents can negotiate deals to exchange resources in order to improve their respective welfare. An example would be: "I give you r_1 if you give me r_2 and r_3". In the most general case, any numbers of agents and resources could be involved in a single deal. From an abstract point of view, a deal takes us from one allocation of resources to the next. In that sense, we may characterise a *deal* as a pair of allocations $\delta = (A, A')$ with $A \neq A'$. That is, deal δ is only applicable in situation A and will result in situation A'. It specifies for each resource in the system whether it is to remain where it is or where it is to be moved to, respectively.

Less formally, we will, for instance, speak about the 'deal' whereby agent i hands resource r over to agent j. In fact, such a transaction really identifies the *class* of all deals that involve resource r being moved from agent i to j while all other resources remain where they are.

Individual rationality. Our agents are self-interested in the sense that they will only propose or accept deals that strictly increase their individual welfare. We call a deal individually rational iff it increases the welfare of all the agents involved in it. A deal may be accompanied by a payment to compensate some of the partners for accepting a loss in utility. Rather than specifying for each pair of agents how much money the former pays to the latter, we simply say how much money each single agent pays or receives. Formally, a *payment function* p is a function from $\mathcal{A}$ to $\mathbb{R}$ such that $\sum_{i \in \mathcal{A}} p(i) = 0$. Here, $p(i) > 0$ means that agent i pays the amount of $p(i)$, while $p(i) < 0$ means that it receives the amount of $-p(i)$.

We can now give a formal definition of individual rationality: A deal $\delta = (A, A')$ is *individually rational* iff there exists a payment function p such that $u_i(A') - u_i(A) > p(i)$ for all $i \in \mathcal{A}$, except possibly $p(i) = 0$ for agents i with $A(i) = A'(i)$ (i.e. for agents not affected by the deal). That is, agent i will be prepared to accept δ iff it has to pay less than its gain in utility or iff it will get paid more than its loss in utility, respectively. Usually, there will be a range of possible payments. How agents manage to agree on a particular one is not a matter of consideration at the abstract level at which we are discussing this

[1] As the amount of money present in the system stays constant throughout the negotiation process, it makes sense not to take it into account for the evaluation of social welfare.

framework here. We assume that a deal will go ahead as long as there exists *some* suitable payment function p.

Types of deals. Following Sandholm [5], we can distinguish a number of *deal types*. The simplest deals are *one-resource-at-a-time deals* where a single resource is passed on from one agent to another one. Deals where one agent passes a set of resources on to another agent are called *cluster* deals. The following example shows that one-resource-at-a-time deals alone are not always sufficient to guarantee the optimal outcome of a negotiation:

Agent 1	Agent 2
$A_0(1) = \{r_1, r_2\}$	$A_0(2) = \{\,\}$
$u_1(\{\,\}) = 0$	$u_2(\{\,\}) = 0$
$u_1(\{r_1\}) = 2$	$u_2(\{r_1\}) = 3$
$u_1(\{r_2\}) = 3$	$u_2(\{r_2\}) = 3$
$u_1(\{r_1, r_2\}) = 7$	$u_2(\{r_1, r_2\}) = 8$

Social welfare for the initial allocation A_0 is 7, but it could be 8, namely if agent 2 had both resources. However, the only possible one-resource-at-a-time deals would be to pass either r_1 or r_2 from agent 1 to agent 2. In either case, the loss in utility incurred by agent 1 (5 or 4, respectively) would outweigh the gain of agent 2 (3 for either deal), so there is no payment function that would make these deals individually rational. The cluster deal of passing $\{r_1, r_2\}$ from agent 1 to 2, on the other hand, would be individually rational if agent 2 paid agent 1 an amount of, say, 7.5.

Deals where one agent gives a single item to another agent who returns another single item are called *swap deals*. Sometimes it can also be necessary to exchange resources between more than just two agents. A *multiagent deal* is a deal that could involve any number of agents, where each agent passes at most one resource to each of the other agents taking part. Similarly to the example above, we can also construct scenarios where swap deals or multiagent deals are necessary (i.e. where cluster deals alone would not be sufficient to guarantee maximal social welfare). This also follows from Theorem 2, which we are going to prove in the next section.

Finally, deals that combine the features of the cluster and the multiagent deal type are called *combined deals*. These could involve any number of agents and any number of resources. In other words, *every* deal δ (according to our abstract definition given above) is a combined deal.

3 Resource Allocation with Money

In this section, we rephrase the main results obtained by Sandholm [5] (on necessary and sufficient contract types in task-oriented domains) using the terminology of our resource allocation problems. The following lemma, which states that a deal is individually rational iff it increases social welfare, will simplify the proofs of the subsequent theorems.

Lemma 1 (Individual rationality and social welfare) *A deal $\delta = (A, A')$ is individually rational iff $sw(A) < sw(A')$.*

Proof. '$\Rightarrow$': By definition, $\delta = (A, A')$ is individually rational iff there exists a payment function p with $u_i(A') - u_i(A) > p(i)$ for all $i \in \mathcal{A}$, except possibly $p(i) = 0$ in case $A(i) = A'(i)$. If we add up the inequations for all $i \in \mathcal{A}$ we get: $\sum_{i\in\mathcal{A}}(u_i(A') - u_i(A)) > \sum_{i\in\mathcal{A}} p(i)$. By definition of a payment function, the righthand side equates to 0 while, by definition of social welfare, the lefthand side equals $sw(A') - sw(A)$, i.e. we really get $sw(A) < sw(A')$ as claimed.

'$\Leftarrow$': Now let $sw(A) < sw(A')$. We are done if we can show that there exists a payment function p such that $u_i(A') - u_i(A) > p(i)$ for all $i \in \mathcal{A}$. The function p with $p(i) = u_i(A') - u_i(A) - \frac{sw(A') - sw(A)}{|\mathcal{A}|}$ meets this requirement (because of $sw(A) < sw(A')$) and $\sum_{i\in\mathcal{A}} p(i) = 0$, i.e. p really is a payment function. □

Sufficient deal types. The following theorem is equivalent to Sandholm's result regarding the *sufficiency* of the combined contract type [5, Prop. 10].

Theorem 1 (Sufficient deals for maximal social welfare) *Any sequence of combined deals (with money) that are individually rational will culminate in a resource allocation with maximal social welfare.*

Proof. By Lemma 1, any individually rational deal will strictly increase social welfare. Hence, as the number of distinct allocations is finite, negotiation will terminate after a finite number of deals. Now, for the sake of contradiction, assume negotiation terminates with a non-optimal allocation A, i.e. there exists another allocation A' with $sw(A) < sw(A')$. But then, by Lemma 1, the deal $\delta = (A, A')$ would be individually rational and thereby possible, which contradicts our assumption of A being a terminal allocation. □

At first sight, this result may seem almost trivial. The notion of a combined deal is a *very* powerful one. A single deal of this type allows for any number of resources to be moved between any number of agents. From this point of view, it is not particularly surprising that we can always reach an optimal allocation (even in just a single step!). Furthermore, *finding* a suitable combined deal is a very complex task, which may not always be viable in practice. But the true power of Theorem 1 is in the fine print: *any* sequence of deals will culminate in an optimal allocation. That is, whatever deals are agreed on in the early stages of the negotiation, the system will never get stuck in a local optimum and finding an allocation with maximal social welfare remains an option throughout. Given the restriction to deals that are individually rational for all the agents involved, social welfare must increase with every single deal. Therefore, negotiation always pays off, even if it has to stop early due to computational limitations. (Andersson and Sandholm [1] have conducted a number of experiments on the sequencing of certain contract/deal types to obtain the best possible allocations within a limited amount of time.)

Necessary deal types. The following theorem is equivalent to Sandholm's main result regarding *necessary* contract types [5, Prop. 11]. All other findings on the insufficiency of certain types of contracts reported in [5] may be considered corollaries to this.

Theorem 2 (Necessary deals for maximal social welfare) *Let the sets of agents and resources be fixed. Then for every deal δ there is a resource allocation problem with money such that δ is necessary to reach a resource allocation with maximal social welfare.*

Proof. Given a set of agents $\mathcal{A}$ and a set of resources $\mathcal{R}$, let $\delta = (A, A')$ with $A \neq A'$ be any deal for this system. We need to show that there are a collection of utility functions $\mathcal{U}$ and an initial allocation such that δ is necessary to reach an allocation with maximal social welfare. This would be the case if A' had maximal social welfare, A had the second highest social welfare, and A was the initial allocation of resources. As we have $A \neq A'$, there must be an agent $j \in \mathcal{A}$ such that $A(j) \neq A'(j)$. We now fix utility functions u_i for agents $i \in \mathcal{A}$ and sets of resources $R \subseteq \mathcal{R}$ as follows:

$$u_i(R) \quad = \quad \begin{cases} 2 & \text{if } R = A'(i) \text{ or } (R = A(i) \text{ and } i \neq j) \\ 1 & \text{if } R = A(i) \text{ and } i = j \\ 0 & \text{otherwise} \end{cases}$$

We get $sw(A') = 2 \cdot |\mathcal{A}|$, $sw(A) = sw(A') - 1$, and $sw(B) < sw(A)$ for any other allocation B. That is, A' is the (unique) allocation with maximal social welfare and the only allocation with higher social welfare than A. Therefore, if we make A the initial allocation then $\delta = (A, A')$ would be the only deal increasing social welfare. By Lemma 1, this means that δ would be the only individually rational (and thereby the only possible) deal. Hence, δ is indeed necessary to achieve maximal social welfare. □

Unlimited money. An implicit assumption made in the framework that we have presented so far is that every agent has got an 'unlimited' amount of money available to it to be able to pay other agents whenever this is required for a deal that would increase social welfare. Concretely, if A is the initial allocation and A' is the allocation with maximal social welfare, then agent i may require an amount of money just below the difference $u_i(A') - u_i(A)$ to be able to get through the negotiation process. In the context of task contracting, for which this framework has been proposed originally [5], this may be justifiable, at least if we are mostly interested in the reallocation of tasks and consider 'money' merely a convenient way of keeping track of the utility transfers between friendly agents. For resource allocation problems, on the other hand, it seems questionable to make assumptions about the unlimited availability of one particular resource, namely money.

4 Resource Allocation without Money

As argued before, making assumptions about the unlimited availability of money to compensate other agents for disadvantageous deals is not realistic in all application domains. In this section, we investigate, to what extent the theoretical results of [5], which we have reproduced above, still apply for resource allocation problems *without* money. (This is what Rosenschein and Zlotkin call negotiation without "explicit utility transfer" [3].)

In scenarios without money, that is, if we do not allow for compensatory payments, we cannot always guarantee an outcome with maximal social welfare. This is, for instance, the case for the following simple problem:

Agent 1	Agent 2
$A_0(1) = \{r\}$	$A_0(2) = \{\,\}$
$u_1(\{\,\}) = 0$	$u_2(\{\,\}) = 0$
$u_1(\{r\}) = 4$	$u_2(\{r\}) = 7$

Here, passing r from agent 1 to agent 2 would increase social welfare by an amount of 3. For the framework *with* money, agent 2 could pay agent 1, say, the amount of 5.5 and the deal would be individually rational for both of them. Without money, however, no deal is possible and negotiation must terminate with a non-optimal allocation.

Pareto optimality. As maximising social welfare is not generally possible, the next best thing would be to investigate whether a Pareto optimal outcome is possible in the framework without money, and what types of deals are sufficient to guarantee this. In this context, we call A a *Pareto optimal* allocation iff there is no other allocation where social welfare is higher while no single agent has lower utility,[2] that is, iff there exists no A' such that $sw(A) < sw(A')$ and $u_i(A) \leq u_i(A')$ for all $i \in \mathcal{A}$.

Individual rationality revisited. To get a sufficiency result, we need to relax the notion of individual rationality a little. For scenarios without money, we now also want agents to agree to a deal, if this at least maintains their utility (i.e. no strict increase is necessary). However, we still require at least one agent (say, the one proposing the deal) to strictly increase their utility. We call deals of this type *cooperative-individually rational*, i.e. $\delta = (A, A')$ is cooperative-individually rational iff $u_i(A) \leq u_i(A')$ for all $i \in \mathcal{A}$ and there exists a $j \in \mathcal{A}$ such that $u_j(A) < u_j(A')$.

Observe that, in analogy to Lemma 1, we still have $sw(A) < sw(A')$ for any cooperative-individually rational deal $\delta = (A, A')$, but *not* vice versa.

[2] This formulation is equivalent to the more common one: "An agreement is Pareto optimal if there is no other agreement [. . .] that is better for some of the agents and not worse for the others." (quoted after [2]).

Sufficient deal types. The following result mirrors Theorem 1.

Theorem 3 (Sufficient deals for Pareto optimality) *Any sequence of combined deals (without money) that are cooperative-individually rational will culminate in a Pareto optimal allocation of resources.*

Proof. Every cooperative-individually rational deal strictly increases social welfare.[3] Together with the fact that there are only finitely many allocations, this implies that any negotiation will eventually terminate. For the sake of contradiction, assume negotiation ends with allocation A, but A is not Pareto optimal. The latter means that there exists another allocation A' with $sw(A) < sw(A')$ and $u_i(A) \leq u_i(A')$ for all $i \in \mathcal{A}$. If we had $u_i(A) = u_i(A')$ for all $i \in \mathcal{A}$, then also $sw(A) = sw(A')$, that is, there must be at least one $j \in \mathcal{A}$ with $u_j(A) < u_j(A')$. But then the deal $\delta = (A, A')$ would be cooperative-individually rational, which contradicts our assumption of A being a terminal allocation. □

Necessary deal types. As the following theorem shows, also for the framework without money, each and every deal may be necessary in order to be able to guarantee an optimal outcome of a negotiation (cf. Theorem 2).

Theorem 4 (Necessary deals for Pareto optimality) *Let the sets of agents and resources be fixed. Then for every deal δ there is a resource allocation problem without money such that δ is necessary to reach a Pareto optimal allocation of resources.*

Proof. Let $\delta = (A, A')$ with $A \neq A'$. We try to fix utility functions u_i in such a way that A' has the highest and A has the second highest social welfare, and that $u_i(A) \leq u_i(A')$ for all agents $i \in \mathcal{A}$. As we have $A \neq A'$, there must be a $j \in \mathcal{A}$ such that $A(j) \neq A'(j)$. We now define utility functions as follows:

$$u_i(R) \quad = \quad \begin{cases} 2 & \text{if } R = A'(i) \text{ or } (R = A(i) \text{ and } i \neq j) \\ 1 & \text{if } R = A(i) \text{ and } i = j \\ 0 & \text{otherwise} \end{cases}$$

We get $sw(A') = 2 \cdot |\mathcal{A}|$, $sw(A) = sw(A') - 1$, and $sw(B) < sw(A)$ for all other allocations B. We also have $u_i(A) \leq u_i(A')$ for all $i \in \mathcal{A}$. Hence, A is not Pareto optimal, but A' is. If we make A the initial allocation, then δ would be the only cooperative-individually rational deal (as every other deal would decrease social welfare), i.e. δ is indeed necessary to guarantee a Pareto optimal outcome. □

Observe that, while this proof has been very similar to the proof of Theorem 2, now we also required the additional condition of $u_i(A) \leq u_i(A')$ for all $i \in \mathcal{A}$.

[3] This is where we need the condition that at least one agent behaves *truly* individually rational for each deal.

5 Specific Utility Functions

In Section 2 we have defined utility functions as (arbitrary) functions from $2^{\mathcal{R}}$ to $\mathbb{R}$. For many application domains this degree of generality may be inappropriate and we may be able to obtain stronger results for specific classes of utility functions. In this section, we discuss some examples.

Clearly, the results on the sufficiency of the combined deal type (Theorems 1 and 3) will still apply, whatever restrictions we may put on utility functions. Interesting new results would be either that the combined deal type is still necessary, even for a restricted class of utility functions, or that a weaker deal type is sufficient for certain domains.

Non-negative and monotone utility. In general, there may be certain resources we would like to assign a negative utility to (e.g. 'five tons of radioactive waste'), but in many domains *non-negative* utility functions with $u_i(R) \geq 0$ for all $R \subseteq \mathcal{R}$ will suffice. All results on the necessity of deals still apply for scenarios with non-negative utility functions (see proofs of Theorems 2 and 4).

A slightly stronger restriction would be to assign at least a small positive value to every non-empty set of resources. We define *positive* utility functions as non-negative functions with $u_i(R) = 0$ iff $R = \{\,\}$. For positive utility functions, Theorem 4 does *not* hold anymore, because any deal that would involve a particular agent giving away all its resources without receiving anything in return could never be cooperative-individually rational (so it could not be necessary either).

Another natural class of utility functions to consider would be the class of *monotone* functions where $R_1 \subset R_2$ implies $u_i(R_1) < u_i(R_2)$. This would be appropriate for domains where every additional resource is known to increase utility at least a bit.

Additive utility. We call a a utility function u_i *additive* iff the value ascribed to a set of resources is always the sum of the values of the single resources in that set, i.e. iff we have:

$$u_i(R_1 \cup R_2) \quad = \quad u_i(R_1) + u_i(R_2) - u_i(R_1 \cap R_2)$$

For domains with additive utility functions, the simple one-resource-at-a-time deal type is sufficient to guarantee outcomes with maximal social welfare in the framework with money.[4]

Theorem 5 (Sufficient deals in additive scenarios) *If the utility functions of all agents are additive, then any sequence of one-resource-at-a-time deals (with money) that are individually rational will culminate in a resource allocation with maximal social welfare.*

[4]This has also been observed by T. Sandholm (personal communication).

Proof. Termination is shown as for Theorem 1. We have to show that whenever the current allocation A does not have maximal social welfare, then there still exists a one-resource-at-a-time deal that is individually rational. In additive domains we have $sw(A) = \sum_{r \in \mathcal{R}} u_{f_A(r)}(\{r\})$ where f_A is the function mapping every resource $r \in \mathcal{R}$ to the agent $i \in \mathcal{A}$ that holds r for allocation A (i.e. $r \in A(i)$). So if there is another allocation A' with $sw(A) < sw(A')$ then there must be at least one resource $r \in \mathcal{R}$ with $u_{f_A(r)}(\{r\}) < u_{f_{A'}(r)}(\{r\})$. But then passing r from agent $f_A(r)$ on to agent $f_{A'}(r)$ would be a one-resource-at-a-time deal that is individually rational, i.e. A cannot be terminal. □

0-1 utility. We call an additive utility function *0-1* iff every single resource has either utility 0 or 1. This may be sufficient if we simply wish to distinguish whether or not an agent *needs* a particular resource (to execute a given plan, for example). This is, for instance, the case for some of the agents defined in [4]. As the following theorem shows, for this kind of domain, the one-resource-at-a-time deal type is even sufficient to guarantee maximal social welfare in the framework without money.

Theorem 6 (Sufficient deals in 0-1 scenarios) *If the utility functions of all agents are 0-1, then any sequence of one-resource-at-a-time deals (without money) that are cooperative-individually rational will culminate in a resource allocation with maximal social welfare.*

Proof. Termination is shown as for Theorem 3. If an allocation A does not have maximal social welfare then it must be the case that an agent i holds a resource r with $u_i(\{r\}) = 0$ and there is another agent j in the system with $u_j(\{r\}) = 1$. Passing r from i to j would be a cooperative-individually rational deal, so either negotiation has not yet terminated or we are in a situation with maximal social welfare. □

Global utility. As a final remark in this section, we point out that we may also use appropriate utility functions to model (limited amounts of) money explicitly. This can be achieved by forcing the utility functions of all the agents in the system to have the same *global* value for certain sets of resources, namely those that represent money.

6 Conclusions

Theoretical results. We believe that the main contribution of this paper lies in the transfer of Sandholm's results on necessary and sufficient conditions for optimal outcomes in negotiation scenarios *with* money (as reported in [5]) to a framework *without* money. This involved replacing the notion of (strict) individual rationality with the notion of cooperative-individual rationality, and the optimality criterion of maximal social welfare with the weaker concept of Pareto optimality. The technical results here are Theorems 3 and 4 on the sufficiency of combined deals and the necessity of all deals, respectively.

The results reported in Section 5 on sufficient deal types for domains with very specific utility functions are rather basic at this stage. Still, we believe, that this could be a very fruitful area for future research. At the moment, theoretical results fall into two extremes: On the one hand, we know that in the general case only the very powerful combined deals are sufficient to guarantee optimal outcomes. On the other hand, we have examples for specific scenarios where the very simple one-resource-at-a-time deal type is sufficient. The truly exciting results would lie somewhere in between: Is there a class of utility functions for which cluster deals (or multiagent deals) are sufficient (and necessary)? Or the other way round: given a class of deals, for what kind of domains (i.e. utility functions) will this class be sufficient to guarantee optimal outcomes?

Protocol design. While the results reported here are rather abstract, they can still have very practical implications. They may, for instance, provide guidelines for the design of concrete protocols for reallocative negotiation scenarios. For example, if the application domain in question can be modelled in terms of additive or even 0-1 utility functions, then Theorems 5 and 6 tell us that it would be inappropriate to allow for dialogue moves for proposing, say, swap deals. At the other end of the spectrum, for domains where we cannot make any strong assumptions on the nature of utility functions, Theorems 2 and 4 show that, ideally, a good protocol should enable agents to agree on *any* kind of deal.

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